

Model Question Paper-03

Physical Sciences
Test Booklet

Sr. No _____
Subject Code: 3

Time: 3:00 Hours

Maximum Marks: 200

INSTRUCTIONS

1. **You have opted for English as medium of Question Paper.** This Test Booklet contains one hundred and forty five (20 Part ‘A’ + 25 Part ‘B’ + 30 Part ‘C’) Multiple Choice Questions (MCQs). You are required to answer a maximum of 15, 20 and 20 questions form Part ‘A’ ‘B’ and ‘C’ respectively. If more than required number of questions are answered, only first 15, 20 and 20 questions in Part ‘A’ ‘B’ and ‘C’ respectively, will be taken up for evaluation.
2. Answer sheet has been provided separately. Before you start filling up your particulars, please ensure that the booklet contains requisite number of pages and that these are not torn or mutilated. If it is so, you may request the Invigilator to change the booklet. Likewise, check the answer sheet also. Sheets to rough work have been appended to the test booklet.
3. Write your Roll No., name, Your address and Serial Number of this Test booklet on the Answer sheet in the space provided on the side 1 of Answer sheet. Also put your signatures in the space identified.
4. **You must darken the appropriate circles with a pencil related to Roll Number, Subject Code, Booklet Code and Center Code on the OMR answer sheet. It is the sole responsibility of the candidate to meticulously follow the instructions given on Answer Sheet, failing which, the computer shall not be able to decipher the correct detail which may ultimately result in loss, including rejection of the OMR answer sheet.**
5. Each question in part ‘A’ carries 2 marks, Part ‘B’ 3.5 marks and Part ‘C’ 5 marks respectively. There will be negative marking @ 25% for each wrong answer.
6. Below each question in Part ‘A’, ‘B’ and ‘C’ four alternatives or responses are given. Only one of these alternatives is the “correct” option to the question. You have to find, for each question, the correct or the best answer.
7. Candidates found copying or resorting to any unfair means are liable to be disqualified from this and future examinations.
8. Candidates should not write anything anywhere except on answer sheet or sheets for rough work.
9. After the test is over, you MUST hand over the Test Booklet and the answer sheet (OMR) to the invigilator.
10. Use of calculator is not permitted.

Roll No.....

I have verified all the information
filled in by the candidate

Name:.....

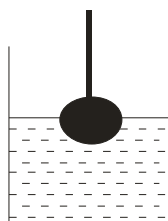
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Signature of the Invigilator

PART-A

1. There is an equilateral triangle in the XY plane with its centre at the origin. The distance of its sides from the origin is 3.5 cm. The area of its circumcircle in cm^2 is

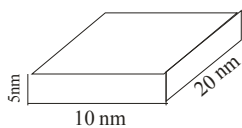
- 1. 38.5
- 2. 49
- 3. 63.65
- 4. 154

2. A sphere of iron of radius $R/2$ fixed to one end of a string was lowered into water in a cylindrical container of base radius R to keep exactly half the sphere dipped. The rise in the level of water in the container will be

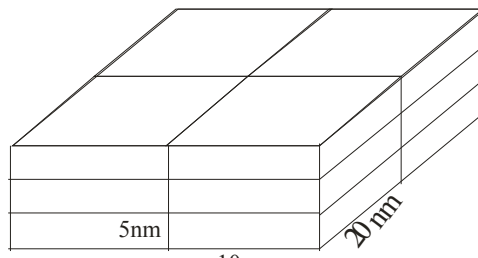


- 1. $R/3$
- 2. $R/4$
- 3. $R/8$
- 4. $R/12$

3. A crystal grows by stacking of unit cells of $10 \times 20 \times 5$ nm size as shown in the diagram given below. How many unit cells will make a crystal of 1cm^3 volume?



Unit Cell (not to scale)



Crystal (not to scale)

- 1. 10^6
- 2. 10^9
- 3. 10^{12}
- 4. 10^{18}

4. What is the value of $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots$ to ∞ ?

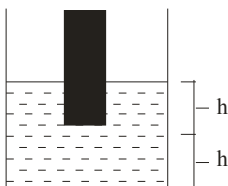
1. $2/3$

2. 1

3. 2

4. ∞

5. A solid cylinder of basal area A was held dipped in water in a cylindrical vessel of basal area $2A$ vertically such that a length h of the cylinder is immersed. The lower tip of the cylinder is at a height h from the base of the vessel. What will be the height of water in the vessel when the cylinder is taken out?



1. $2h$

2. $\frac{3}{2}h$

3. $\frac{4}{3}h$

4. $\frac{5}{4}h$

6. Of all the triangles that can be inscribed in a semicircle of radius R with the diameter as one side, the biggest one has the area

1. R^2

2. $R^2\sqrt{2}$

3. $R^2\sqrt{3}$

4. $2R^2$

7. Choose the largest number:

1. 2^{500}

2. 3^{400}

3. 4^{300}

4. 5^{200}

8. A daily sheet calendar of the year 2013 contains sheets of 10×10 cm size. All the sheets of the calendar are spread over the floor of a room of $5 \text{ m} \times 7.3 \text{ m}$ size. What percentage of the floor will be covered by these sheets?

1. 0.1

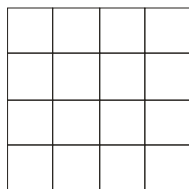
2. 1

3. 10

4. 100

3

9. How many rectangles (which are not squares) are there in the following figure?



1. 56
2. 70
3. 86
4. 100

10. Define $a \otimes b = lcm(a,b) + gcd(a,b)$ and $a \oplus b = a^b + b^a$. What is the value of $(1 \oplus 2) \otimes (3 \oplus 4)$? Here lcm = least common multiple and gcd = greatest common divisor.

1. 145
2. 286
3. 436
4. 572

11. A square pyramid is to be made using a wire such that only one strand of wire is used for each edge. What is the minimum number of times that the wire has to be cut in order to make the pyramid?

1. 3
2. 7
3. 2
4. 1

12. A crow is flying along a horizontal circle of radius R at a height R above the horizontal ground. Each of a number of men on the ground found that the angular height of the crow was fixed angle θ ($< 45^\circ$) when it was closet to him. Then all these men must be on a circle on the ground with a radius

1. $R + R \sin \theta$
2. $R + R \cos \theta$
3. $R + R \tan \theta$
4. $R + R \cot \theta$

13. How many pairs of positive integers have gcd 20 and lcm 600?

(gcd = greatest common divisor; lcm = least common multiple)

1. 4
2. 0
3. 1
4. 7

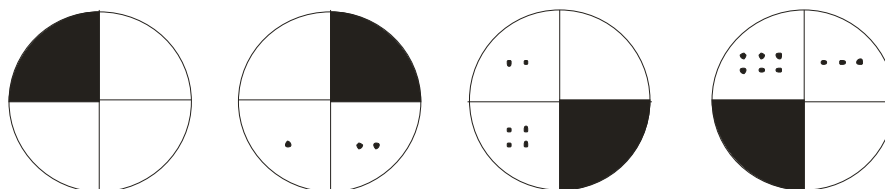
14. During an evening party, when Ms. Black, Ms. Brown and Ms. White met, Ms. Brown remarked, "It is interesting that our dresses are white, black or brown, but for each of us the name does not match the colour of the dress!". Ms. White replied, "But your white dress does not suit you". Pick the correct answer.

- 1. Ms. White's dress was brown
- 2. Ms. Black's dress was white.
- 3. Ms. White's dress was black
- 4. Ms. Black's dress was black.

15. Two integers are picked at random from the first 15 positive integers without replacement. What is the probability that the sum of the two numbers is 20?

- 1. $\frac{3}{4}$
- 2. $\frac{1}{21}$
- 3. $\frac{1}{105}$
- 4. $\frac{1}{20}$

16. Identify the next figure in the sequence



- 1.
- 2.
- 3.
- 4.

Figure

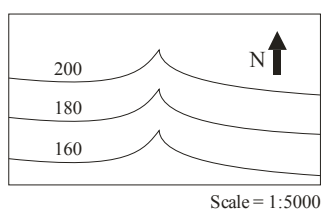
17. In a customer survey conducted during Monday to Friday, of the customers who asked for child care facilities in super markets, 23% were men and the rest, women. Among them, 19.9% of the women and 8.8% of the men were willing to pay for the facilities?

- A. What is the ratio of the men to women customers who wanted child care facilities?

B. If the survey had been conducted during the weekend instead, how will the result change?

- 1. Only A can be answered
- 2. Only B can be answered
- 3. Both A and B can be answered
- 4. Neither A nor B can be answered

18. The map given below shows contour lines which connect points of equal ground surface elevation in a region. Inverted 'V' shaped portions of contour lines represent a valley along which a river flows. What is the downstream direction of the river?

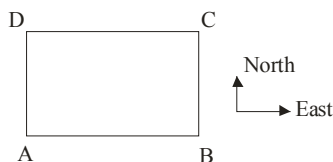


- 1. North
- 2. South
- 3. East
- 4. West

19. During a summer vacation, of 20 friends from a hostel, each wrote a letter to each of all others. The total number of letter written was

- 1. 20
- 2. 400
- 3. 200
- 4. 380

20. A person has to cross a square field by going from A to C. The person is only allowed to move towards the east or towards the north or use a combination of these movements. The total distance travelled by the person.



- 1. depends on the length of each step
- 2. depends on the total number of steps
- 3. is different for different paths
- 4. is the same for all paths

PART-B

21. Two identical bosons of mass m are placed in a one-dimensional potential $V(x) = \frac{1}{2}m\omega^2x^2$.

The bosons interact via a weak potential,

$$V_{12} = V_0 \exp[-m\Omega(x_1 - x_2)^2 / 4\hbar]$$

where x_1 and x_2 denote coordinates of the particles. Given that the ground state wave-

function of the harmonic oscillator is $\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega x^2}{2\hbar}}$. The ground state energy of the two-

boson system, to the first order in V_0 , is:

- | | |
|--|--|
| 1. $\hbar\omega + 2V_0$ | 2. $\hbar\omega + \frac{V_0\Omega}{\omega}$ |
| 3. $\hbar\omega + V_0\left(1 + \frac{\Omega}{2\omega}\right)^{-1/2}$ | 4. $\hbar\omega + V_0\left(1 + \frac{\omega}{\Omega}\right)$ |

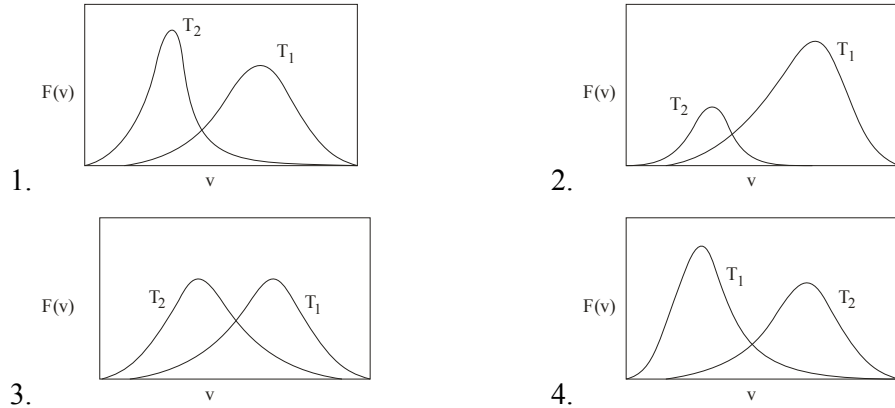
22. Ten grams of ice at 0°C is added to a beaker containing 30 grams of water at 25°C . What is the final temperature of the system when it comes to thermal equilibrium? (The specific heat of water is $1\text{cal/cm}^\circ\text{C}$ and latent heat of melting of ice is 80 cal/gm .)

- | | |
|-------------------------|--------------------------|
| 1. 0°C | 2. 7.5°C |
| 3. 12.5°C | 4. -1.25°C |

23. A vessel has two compartments of volume V_1 and V_2 , containing an ideal gas at pressures p_1 and p_2 , and temperatures T_1 and T_2 respectively. If the wall separating the compartments is removed, the resulting equilibrium temperature will be

- | | |
|--|--|
| 1. $\frac{p_1T_1 + p_2T_2}{p_1 + p_2}$ | 2. $\frac{V_1T_1 + V_2T_2}{V_1 + V_2}$ |
| 3. $\frac{p_1V_1 + p_2V_2}{(p_1V_1/T_1) + (p_2V_2/T_2)}$ | 4. $(T_1T_2)^{1/2}$ |

24. For temperature $T_1 > T_2$, the qualitative temperature dependence of the probability distribution $F(v)$ of the speed v of a molecule in three dimensions is correctly represented by the following figure:



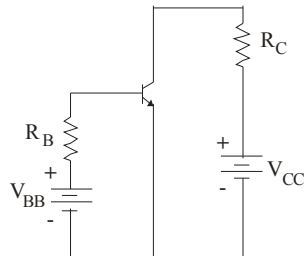
25. A system of non-interacting spin $-1/2$ charged particles are placed in an external magnetic field. At low temperature T , the leading behaviour of the excess energy above the ground state energy, depends on T as: (c is constant)

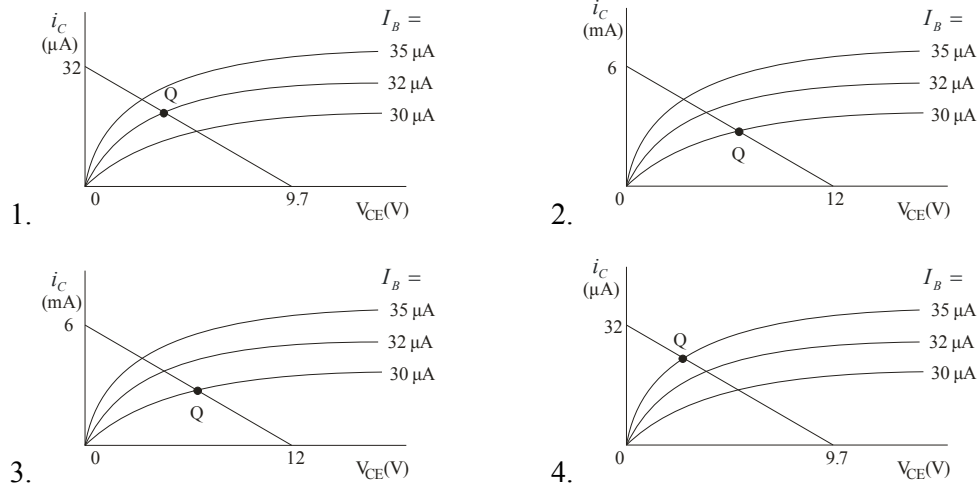
1. cT
2. cT^3
3. $e^{-c/T}$
4. c (is independent of T)

26. The acceleration due to gravity g is determined by measuring the time period T and the length L of a simple pendulum. If the uncertainties in the measurements of T and L are ΔT and ΔL respectively, the fractional error $\Delta g / g$ in measuring g is best approximated by

1. $\frac{|\Delta L|}{L} + \frac{|\Delta T|}{T}$
2. $\frac{|\Delta L|}{L} + \frac{|2\Delta T|}{T}$
3. $\sqrt{\frac{|\Delta L|^2}{L^2} + \frac{|\Delta T|^2}{T^2}}$
4. $\sqrt{\left(\frac{|\Delta L|}{L}\right)^2 + \left(\frac{|2\Delta T|}{T}\right)^2}$

27. A silicon transistor with built-in voltage $0.7V$ is used in the circuit shown, with $V_{BB} = 9.7V$, $R_B = 300 k\Omega$, $V_{CC} = 12V$ and $R_C = 2 k\Omega$. Which of the following figures correctly represents the load line and the quiescent Q point?

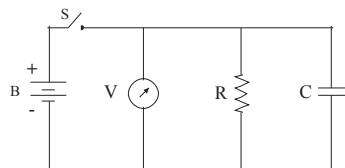




28. If the analog input to an 8-bit successive approximation ADC is increased from 1.0 V to 2.0 V, then the conversion time will

- 1. remain unchanged
- 2. double
- 3. decrease to half its original value
- 4. increase four times

29. The insulation resistance R of an insulated cable is measured by connecting it in parallel with a capacitor C, a voltmeter, and battery B as shown. The voltage across the cable dropped from 150 V to 15 V, 1000 seconds after the switch S is closed. If the capacitance of the cable is 5μF then its insulation resistance is approximately



- 1. $10^9 \Omega$
- 2. $10^8 \Omega$
- 3. $10^7 \Omega$
- 4. $10^6 \Omega$

30. The approximation $\cos \theta \approx 1$ is valid up to 3 decimal places as long as $|\theta|$ is less than:

(take $180^\circ / \pi \approx 57.29^\circ$)

- 1. 1.28°
- 2. 1.81°
- 3. 3.28°
- 4. 4.01°

31. The area of a disc in its rest frame S is equal to 1 (in some units). The disc will appear distorted to an observer O moving with a speed u with respect to S along the plane of the disc. The area of the disc measured in the rest frame of the observer O is (c is the speed of light in vacuum).

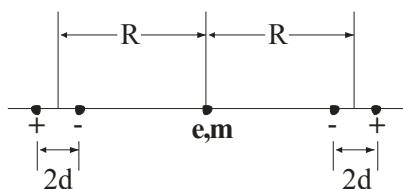
1. $\left(1 - \frac{u^2}{c^2}\right)^{1/2}$

2. $\left(1 - \frac{u^2}{c^2}\right)^{-1/2}$

3. $\left(1 - \frac{u^2}{c^2}\right)$

4. $\left(1 - \frac{u^2}{c^2}\right)^{-1}$

32. A particle of charge e and mass m is located at the midpoint of the line joining two fixed collinear dipoles with unit charges as shown in the figure. (The particle is constrained to move only along the line joining the dipoles). Assuming that the length of the dipoles is much shorter than their separation, the natural frequency of oscillation of the particle is



1. $\sqrt{\frac{6e^2 R^2}{\pi \epsilon_0 m d^5}}$

2. $\sqrt{\frac{6e^2 R}{\pi \epsilon_0 m d^4}}$

3. $\sqrt{\frac{6e^2 R^2}{\pi \epsilon_0 m R^5}}$

4. $\sqrt{\frac{6e^2 d}{\pi \epsilon_0 m R^4}}$

33. A current I is created by a narrow beam of protons moving in vacuum with constant velocity \vec{u} . The direction and magnitude, respectively, of the Poynting vector \vec{S} outside the beam at a radial distance r (much larger than the width of the beam) from the axis, are

1. $\vec{S} \perp \vec{u}$ and $|\vec{S}| = \frac{I^2}{4\pi^2 \epsilon_0 |\vec{u}| r^2}$

2. $\vec{S} \parallel (-\vec{u})$ and $|\vec{S}| = \frac{I^2}{4\pi^2 \epsilon_0 |\vec{u}| r^4}$

3. $\vec{S} \parallel \vec{u}$ and $|\vec{S}| = \frac{I^2}{4\pi^2 \epsilon_0 |\vec{u}| r^2}$

4. $\vec{S} \parallel \vec{u}$ and $|\vec{S}| = \frac{I^2}{4\pi^2 \epsilon_0 |\vec{u}| r^4}$

34. If the electric and magnetic field are unchanged when the vector potential \vec{A} changes (in suitable units) according to $\vec{A} \rightarrow \vec{A} + \hat{r}$, where $\vec{r} = r(t)\hat{r}$, then the scalar potential Φ must simultaneously change to

- | | |
|-------------------------------------|-------------------------------------|
| 1. $\Phi - r$ | 2. $\Phi + r$ |
| 3. $\Phi - \partial r / \partial t$ | 4. $\Phi + \partial r / \partial t$ |

35. Consider an axially symmetric static charge distribution of the form,

$$\rho = \rho_0 \left(\frac{r_0}{r} \right)^2 e^{-r/r_0} \cos^2 \varphi$$

The radial component of the dipole moment due to this charge distribution is

- | | |
|-----------------------|--------------------------|
| 1. $2\pi\rho_0 r_0^4$ | 2. $\pi\rho_0 r_0^4$ |
| 3. $\rho_0 r_0^4$ | 4. $\pi\rho_0 r_0^4 / 2$ |

36. In a basis in which the z -component S_z of the spin is diagonal, an electron is in a spin state

$$\psi = \begin{pmatrix} (1+i)/\sqrt{6} \\ \sqrt{2}/3 \end{pmatrix}$$

The probabilities that a measurement of S_z will yield the values $\hbar/2$ and $-\hbar/2$ are, respectively,

- | | |
|----------------|----------------|
| 1. 1/2 and 1/2 | 2. 2/3 and 1/3 |
| 3. 1/4 and 3/4 | 4. 1/3 and 2/3 |

37. Consider the normalized state $|\psi\rangle$ of a particle in a one-dimensional harmonic oscillator:

$$|\psi\rangle = |b_1/0\rangle + |b_2/1\rangle$$

where $|0\rangle$ and $|1\rangle$ denote the ground and first excited states respectively, and b_1 and b_2 are real constants. The expectation value of the displacement x in the state $|\psi\rangle$ will be a minimum when

- | | |
|----------------------------|-----------------------------------|
| 1. $b_2 = 0, (b_1 = 1)$ | 2. $b_2 = \frac{1}{\sqrt{2}} b_1$ |
| 3. $b_2 = \frac{1}{2} b_1$ | 4. $b_2 = b_1$ |

38. A muon (μ^-) from cosmic rays is trapped by a proton to form a hydrogen-like atom. Given that a muon is approximately 200 times heavier than an electron, the longest wavelength of the spectral line (in the analogue of the Lyman series) of such an atom will be

- | | |
|-----------|-----------|
| 1. 5.62 Å | 2. 6.67 Å |
| 3. 3.75 Å | 4. 13.3 Å |

39. The un-normalized wavefunction of a particle in a spherically symmetric potential is given by $\psi(\vec{r}) = zf(r)$ where $f(r)$ is a function of the radial variable r . The eigenvalue of the operator \vec{L}^2 (namely the square of the orbital angular momentum) is

- | | |
|------------------|------------------|
| 1. $\hbar^2 / 4$ | 2. $\hbar^2 / 2$ |
| 3. \hbar^2 | 4. $2\hbar^2$ |

40. Give that $\sum_{n=0}^{\infty} H_n(x) \frac{t^n}{n!} = e^{-t^2+2tx}$, the value of $H_4(0)$ is

- | | |
|-------|-------|
| 1. 12 | 2. 6 |
| 3. 24 | 4. -6 |

41. A unit vector \hat{n} on the xy -plane is at an angle of 120° with respect to \hat{i} . The angle between the vectors $\vec{u} = a\hat{i} + b\hat{n}$ and $\vec{v} = a\hat{n} + b\hat{i}$ will be 60° if

- | | |
|------------------------|------------------------|
| 1. $b = \sqrt{3}a / 2$ | 2. $b = 2a / \sqrt{3}$ |
| 3. $b = a / 2$ | 4. $b = a$ |

42. With $z = x + iy$, which of the following functions $f(x,y)$ is NOT a (complex) analytic function of z ?

- | | |
|---|-------------------------------------|
| 1. $f(x,y) = (x+iy-8)^3 (4+x^2-y^2+2ixy)^7$ | 2. $f(x,y) = (x+iy)^7 (1-x-iy)^3$ |
| 3. $f(x,y) = (x^2-y^2+2ixy-3)^5$ | 4. $f(x,y) = (1-x+iy)^4 (2+x+iy)^6$ |

43. A planet of mass m and an angular momentum L moves in a circular orbit in a potential, $V(r) = -k/r$, where k is a constant. If it is slightly perturbed radially, the angular frequency of radial oscillations is

- | | |
|-------------------------|-------------------------|
| 1. $mk^2 / \sqrt{2}L^3$ | 2. mk^2 / L^3 |
| 3. $\sqrt{2}mk^2 / L^3$ | 4. $\sqrt{3}mk^2 / L^3$ |

44. The Lagrangian of a particle of mass m moving in one dimension is given by $L = \frac{1}{2}m\dot{x}^2 - bx$ where b is a positive constant. The coordinate of the particle $x(t)$ at time t is given by: (in the following c_1 and c_2 are constant)

- | | |
|--|--|
| 1. $-\frac{b}{2m}t^2 + c_1t + c_2$ | 2. $c_1t + c_2$ |
| 3. $c_1 \cos\left(\frac{bt}{m}\right) + c_2 \sin\left(\frac{bt}{m}\right)$ | 4. $c_1 \cosh\left(\frac{bt}{m}\right) + c_2 \sinh\left(\frac{bt}{m}\right)$ |

45. A uniform cylinder of radius r and length l , and a uniform sphere of radius R are released on an inclined plane when their centres of mass are at the same height. If they roll down without slipping, and if the sphere reaches the bottom of the plane with a speed V , then the speed of the cylinder when it reaches the bottom is:

- | | |
|---------------------------------|-----------------------------|
| 1. $V\sqrt{\frac{14rl}{15R^2}}$ | 2. $4V\sqrt{\frac{r}{15R}}$ |
| 3. $\frac{4V}{\sqrt{15}}$ | 4. $V\sqrt{\frac{14}{15}}$ |

PART-C

46. The components of a vector potential $A_\mu \equiv (A_0, A_1, A_2, A_3)$, are given by $A_\mu = k(-xyz, yzt, zxt, xyt)$ where k is a constant. The three components of the electric field are

1. $k(yz, zx, xy)$
2. $k(x, y, z)$
3. $(0, 0, 0)$
4. $k(xt, yt, zt)$

47. In the Born approximation, the scattering amplitude $f(\theta)$ for the Yukawa potential

$$V(r) = \frac{\beta e^{-\mu r}}{r} \text{ is given by: (in the following } b = 2k \sin \frac{\theta}{2}, E = \hbar^2 k^2 / 2m \text{)}$$

1. $-\frac{2m\beta}{\hbar^2(\mu^2 + b^2)^2}$
2. $-\frac{2m\beta}{\hbar^2(\mu^2 + b^2)}$
3. $-\frac{2m\beta}{\hbar^2 \sqrt{\mu^2 + b^2}}$
4. $-\frac{2m\beta}{\hbar^2(\mu^2 + b^2)^3}$

48. If ψ_{nlm} denotes the eigenfunction of the Hamiltonian with a potential $V = V(r)$ then the expectation value of the operator $L_x^2 + L_y^2$ in the same

$$\psi = \frac{1}{5} [3\psi_{211} + \psi_{210} - \sqrt{15} \psi_{21-1}] \text{ is}$$

1. $39\hbar^2 / 25$
2. $13\hbar^2 / 25$
3. $2\hbar^2$
4. $26\hbar^2 / 25$

49. An oscillating current $I(t) = I_0 \exp(-i\omega t)$ flow in the direction of the y -axis through a thin metal sheet of area 1.0 cm^2 kept in the xy -plane. The rate of total energy radiated per unit area from the surfaces of the metal sheet at a distance of 100 m is

1. $I_0 \omega / (12\pi \epsilon_0 c^3)$
2. $I_0^2 \omega^2 / (12\pi \epsilon_0 c^3)$
3. $I_0^2 \omega / (12\pi \epsilon_0 c^3)$
4. $I_0 \omega^2 / (24\pi \epsilon_0 c^3)$

50. Consider a two-dimensional infinite square well

$$V(x, y) = \begin{cases} 0 & 0 < x < a, \quad 0 < y < a \\ \infty & \text{otherwise} \end{cases}$$

Its normalized eigenfunctions are

$$\psi_{n_x, n_y}(x, y) = \frac{2}{a} \sin\left(\frac{n_x \pi x}{a}\right) \sin\left(\frac{n_y \pi y}{a}\right)$$

where $n_x, n_y = 1, 2, 3, \dots$. If a perturbation

$$H' = \begin{cases} V_0 & 0 < x < \frac{a}{2}, 0 < y < \frac{a}{2} \\ 0 & \text{otherwise} \end{cases}$$

is applied, then the correction to the energy of the first excited state to order V_0 is

- | | |
|---|---|
| 1. $\frac{V_0}{4}$ | 2. $\frac{V_0}{4} \left[1 \pm \frac{64}{9\pi^2} \right]$ |
| 3. $\frac{V_0}{4} \left[1 \pm \frac{16}{9\pi^2} \right]$ | 4. $\frac{V_0}{4} \left[1 \pm \frac{32}{9\pi^2} \right]$ |

51. Consider a system of two Ising spins S_1 and S_2 taking values ± 1 with interaction energy given by $\varepsilon = -J S_1 S_2$, when it is in thermal equilibrium at temperature T . For large T , the average energy of the system varies as $C/k_B T$, with C given by

- | | |
|------------|-----------|
| 1. $-2J^2$ | 2. $-J^2$ |
| 3. J^2 | 4. $4J$ |

52. Consider three particles A, B and C, each with an attribute S that can take two values ± 1 . Let $S_A = 1, S_B = 1$ and $S_C = -1$, at a given instant. In the next instant, each S value can change to $-S$ with probability $1/3$. The probability that $S_A + S_B + S_C$ remains unchanged is

- | | |
|----------|----------|
| 1. $2/3$ | 2. $1/3$ |
| 2. $2/9$ | 4. $4/9$ |

53. The bound on the ground state energy of the Hamiltonian with an attractive delta-function potential, namely

$$H = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} - \alpha \delta(x)$$

using the variational principle with the trial wavefunction $\psi(x) = A \exp(-bx^2)$ is

[Note: $\int_0^\infty e^{-t} t^a dt = \Gamma(a+1)$]

- | | |
|-------------------------------|--------------------------------------|
| 1. $-m\alpha^2 / 4\pi\hbar^2$ | 2. $-m\alpha^2 / 2\pi\hbar^2$ |
| 3. $-m\alpha^2 / \pi\hbar^2$ | 4. $-m\alpha^2 / \sqrt{5}\pi\hbar^2$ |

54. Consider two different systems each with three identical non-interacting particles. Both have single particle states with energies $\varepsilon_0, 3\varepsilon_0$ and $5\varepsilon_0$, ($\varepsilon_0 > 0$). One system is populated by spin $s = 1/2$ fermions and the other by bosons. What is the value of $E_F - E_B$ where E_F and E_B are the ground state energies of the fermionic and bosonic systems respectively?

1. $6\varepsilon_0$
2. $2\varepsilon_0$
3. $4\varepsilon_0$
4. ε_0

55. The input to a look-in amplifier has the form $V_i(t) = V_i \sin(\omega t + \theta_i)$ where V_i, ω, θ_i are the amplitude, frequency and phase of the input signal respectively. This signal is multiplied by a reference signal of the same frequency ω , amplitude V_r and phase θ_r . If the multiplied signal is fed to a low pass filter of cut-off frequency ω , then the final output signal is

1. $\frac{1}{2} V_i V_r \cos(\theta_i - \theta_r)$
2. $\frac{1}{2} V_i V_r \left[\cos(\theta_i - \theta_r) - \cos\left(\frac{1}{2} \omega t + \theta_i + \theta_r\right) \right]$
3. $V_i V_r \sin(\theta_i - \theta_r)$
4. $\frac{1}{2} V_i V_r \left[\cos(\theta_i - \theta_r) + \cos\left(\frac{1}{2} \omega t + \theta_i + \theta_r\right) \right]$

56. The solution of the partial differential equation

$$\frac{\partial^2}{\partial t^2} u(x, t) - \frac{\partial^2}{\partial x^2} u(x, t) = 0$$

Satisfying the boundary conditions $u(0, t) = 0 = u(L, t)$ and initial conditions

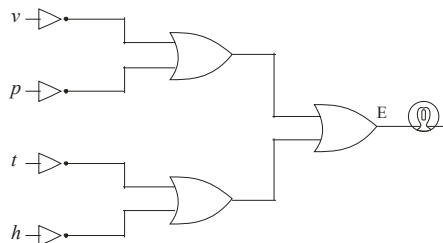
$$u(x, 0) = \sin(\pi x / L) \text{ and } \left. \frac{\partial}{\partial t} u(x, t) \right|_{t=0} = \sin(2\pi x / L) \text{ is}$$

- (1) $\sin(\pi x / L) \cos(\pi t / L) + \frac{L}{2\pi} \sin(2\pi x / L) \cos(2\pi t / L)$
- (2) $2 \sin(\pi x / L) \cos(\pi t / L) - \sin(\pi x / L) \cos(2\pi t / L)$
- (3) $\sin(\pi x / L) \cos(2\pi t / L) + \frac{L}{\pi} \sin(2\pi x / L) \sin(\pi t / L)$
- (4) $\sin(\pi x / L) \cos(\pi t / L) + \frac{L}{2\pi} \sin(2\pi x / L) \sin(2\pi t / L)$

57. Consider the hydrogen-deuterium molecule HD. If the mean distance between the two atoms is 0.08 nm and the mass of the hydrogen atom is $938 \text{ MeV}/c^2$, then the energy difference ΔE between the two lowest rotational states is approximately

- a) 10^{-1} eV
- b) 10^{-2} eV
- c) $2 \times 10^{-2} \text{ eV}$
- d) 10^{-3} eV

58. Four digital outputs V, P, T and H monitor the speed v , tyre pressure p , temperature t and relative humidity h of a car. These outputs switch from 0 to 1 when the values of the parameters exceed 85km/hr, 2 bar 40°C and 50%, respectively. A logic circuit that is used to switch ON a lamp at the output E is shown below.



1. $v < 85 \text{ km/hr}, p < 2 \text{ bar}, t > 40^\circ\text{C}, h > 50\%$
2. $v < 85 \text{ km/hr}, p < 2 \text{ bar}, t > 40^\circ\text{C}, h < 50\%$
3. $v > 85 \text{ km/hr}, p < 2 \text{ bar}, t > 40^\circ\text{C}, h < 50\%$
4. $v > 85 \text{ km/hr}, p < 2 \text{ bar}, t > 40^\circ\text{C}, h > 50\%$

59. The solution of the differential equation $\frac{dx}{dt} = x^2$ with the initial condition $x(0) = 1$ will blow up as t tends to

- | | |
|--------|-------|
| 1. 1 | 2. 2 |
| 3. 1/2 | 4. -1 |

60. Let u be a random variable uniformly distributed in the interval $[0,1]$ and $V = -c \ln u$, where c is a real constant. If V is to be exponentially distributed in the interval $[0, \infty]$ with unit standard deviation, then the value of c should be

- | | |
|------------|--------|
| 1. $\ln 2$ | 2. 1/2 |
| 3. 1 | 4. -1 |

61. The inverse Laplace transform of $\frac{1}{s^2(s+1)}$ is

- | | |
|---------------------------|----------------------------------|
| 1. $\frac{1}{2}t^2e^{-t}$ | 2. $\frac{1}{2}t^2 + 1 - e^{-t}$ |
| 3. $t - 1 + e^{-t}$ | 4. $\frac{1}{2}t^2(1 - e^{-t})$ |

62. The number of degrees of freedom of a rigid body in d space-dimensions is

- | | |
|---------------|---------|
| 1. $2d$ | 2. 6 |
| 3. $d(d+1)/2$ | 4. $d!$ |

63. A particle of mass m is at the stable equilibrium position of its potential energy $V(x) = ax - bx^3$ where a, b are positive constants. The minimum velocity that has to be imparted to the particle to render its motion unstable is

- | | |
|-----------------------------|-----------------------------|
| 1. $(64a^3 / 9m^2b)^{1/4}$ | 2. $(64a^3 / 27m^2b)^{1/4}$ |
| 3. $(16a^3 / 27m^2b)^{1/4}$ | 4. $(3a^3 / 64m^2b)^{1/4}$ |

64. If the operators A and B satisfy the commutation relation $[A, B] = I$, where I is the identity operator, then

- | | |
|-----------------------------|--------------------------|
| 1. $[e^A, B] = e^A$ | 2. $[e^A, B] = [e^B, A]$ |
| 3. $[e^A, B] = [e^{-B}, A]$ | 4. $[e^A, B] = I$ |

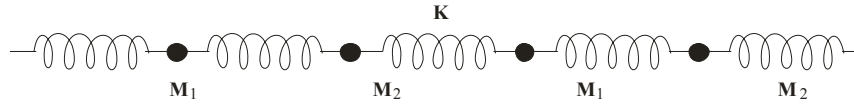
65. A system is governed by the Hamiltonian $H = \frac{1}{2}(p_x - ay)^2 + \frac{1}{2}(p_y - bx)^2$ where a and b are constants and p_x, p_y are momenta conjugate to x and y respectively. For what values of a and b will the quantities $(p_x - 3y)(p_y + 2x)$ be conserved?

- | | |
|--------------------|--------------------|
| 1. $a = -3, b = 2$ | 2. $a = 3, b = -2$ |
| 3. $a = 2, b = -3$ | 4. $a = -2, b = 3$ |

66. Using the frequency-dependent Drude formula, what is the effective kinetic inductance of a metallic wire that is to be used as a transmission line? [In the following, the electron mass is m , density of electrons is n , and the length and cross-sectional area of the wire are l and A respectively].

- | | |
|-------------------|----------------------------|
| 1. $mA / (ne^2l)$ | 2. Zero |
| 3. $ml / (ne^2A)$ | 4. $m\sqrt{A} / (ne^2l^2)$ |

67. The phonon dispersion for the following one-dimensional diatomic lattice with masses M_1 and M_2 (as shown in the figure) is given by



$$\omega^2(q) = K \left(\frac{1}{M_1} + \frac{1}{M_2} \right) \left[1 \pm \sqrt{1 - \frac{4M_1M_2}{(M_1 + M_2)^2} \sin^2 \left(\frac{qa}{2} \right)} \right]$$

where a is the lattice parameter and K is the spring constant. The velocity of sound is

- | | |
|---|---|
| 1. $\sqrt{\frac{K(M_1 + M_2)}{2M_1M_2}}a$ | 2. $\sqrt{\frac{K}{2(M_1 + M_2)}}a$ |
| 3. $\sqrt{\frac{K(M_1 + M_2)}{M_1M_2}}a$ | 4. $\sqrt{\frac{KM_1M_2}{2(M_1 + M_2)^3}}a$ |

68. The binding energy of a light nucleus (Z, A) in MeV is given by the approximate formula

$$B(A, Z) \approx 16A - 20A^{2/3} - \frac{3}{4}Z^2A^{-1/3} + 30 \frac{(N - Z)^2}{A}$$

where $N = A - Z$ is the neutron number. The value of Z of the most stable isobar for a given A is

- | | |
|--|--|
| 1. $\frac{A}{2} \left(1 - \frac{A^{2/3}}{160} \right)^{-1}$ | 2. $\frac{A}{2}$ |
| 3. $\frac{A}{2} \left(1 - \frac{A^{2/3}}{120} \right)^{-1}$ | 4. $\frac{A}{2} \left(1 + \frac{A^{4/3}}{64} \right)$ |

69. Muons are produced through the annihilation of particle a and its antiparticle, namely the process

$$a + \bar{a} \rightarrow \mu^+ + \mu^-$$

A muon has a rest mass of $105 \text{ MeV}/c^2$ and its proper life time is $2 \mu\text{s}$. If the centre of mass energy of the collision is 2.1 GeV in the laboratory frame that coincides with the centre-of-mass frame, then the fraction of muons that will decay before they reach a detector placed 6 km away from the interaction point is

- | | |
|-----------------|-----------------|
| 1. e^{-1} | 2. $1 - e^{-1}$ |
| 3. $1 - e^{-2}$ | 4. e^{-10} |

70. The conductors in a 0.75 km long two-wire transmission line are separated by a centre-of-centre distance of 0.2 m. If each conductor has a diameter of 4 cm, then the capacitance of the line is

- | | |
|-----------------------|-----------------------|
| 1. 8.85 μF | 2. 88.5 μF |
| 3. 8.85 pF | 4. 8.85 nF |

71. The electron dispersion relation for a one-dimensional metal is given by

$$\varepsilon_k = 2\varepsilon_0 \left[\sin^2 \frac{ka}{2} - \frac{1}{6} \sin^2 ka \right]$$

Where k is the momentum, a is the lattice constant, ε_0 is a constant having dimensions of energy and $|ka| \leq \pi$. If the average number of electrons per atom in the conduction band is $1/3$, then the Fermi energy is

- | | |
|-------------------------|-------------------------|
| 1. $\varepsilon_0 / 4$ | 2. ε_0 |
| 3. $2\varepsilon_0 / 3$ | 4. $5\varepsilon_0 / 3$ |

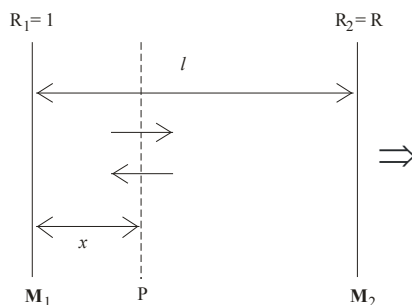
72. The electronic energy levels in a hydrogen atom are given by $E_n = -13.6/n^2 \text{ eV}$. If a selective excitation to the $n = 100$ level is to be made using a laser, the maximum allowed frequency line-width of the laser is

- | | |
|------------|------------|
| 1. 6.5 MHz | 2. 6.5 GHz |
| 3. 6.5 Hz | 4. 6.5 kHz |

73. If the energy dispersion of a two-dimensional electron system is $E = u\hbar k$ where u is the velocity and k is the momentum, then the density of states $D(E)$ depends on the energy as

- | | |
|-----------------|---------------|
| 1. $1/\sqrt{E}$ | 2. \sqrt{E} |
| 3. E | 4. constant |

74. Consider the laser resonator cavity shown in the figure.



If I_1 is the intensity of the radiation at mirror M_1 and α is the gain coefficient of the medium between the mirrors, then the energy density of photons in the plane P at a distance x from M_1 is

- | | |
|--|---------------------------|
| 1. $(I_1/c)e^{\alpha x}$ | 2. $(I_1/c)e^{-\alpha x}$ |
| 3. $(I_1/c)(e^{\alpha x} + e^{-\alpha x})$ | 4. $(I_1/c)e^{2\alpha x}$ |

75. A spin $-\frac{1}{2}$ particle A undergoes the decay $A \rightarrow B + C + D$ where it is known that B and C are also spins $-\frac{1}{2}$ particles. The complete set of allowed values of the spin of the particle D is

- | | |
|--|--|
| 1. $\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, \dots$ | 2. 0, 1 |
| 3. $\frac{1}{2}$ only | 4. $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \dots$ |

